1) Kinematic Constraints

a) SO(2)

b) SO(3)

c) SE(2) Reuleaux

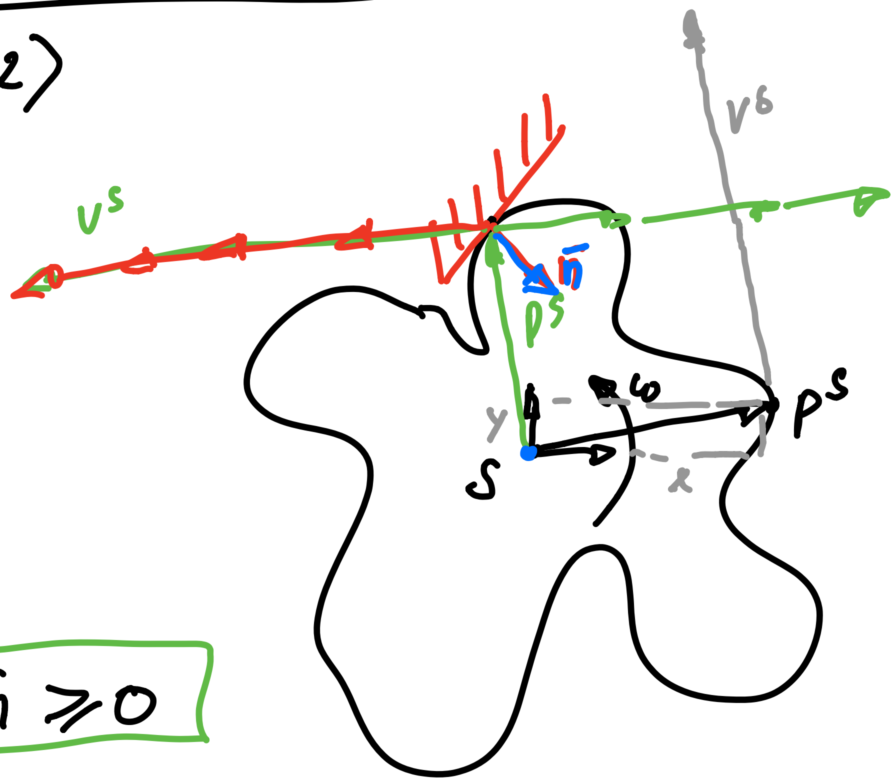
d) SE(3)

2) Friction + Grasp Maps |

Ref:

- M, L & S Murray, Li, Sastry
- L & P Lynch & Park.

① SO(2)



$v^s = \begin{bmatrix} \hat{\omega} & v \\ 0 & 0 \end{bmatrix} p^s$ SE(3)

$$v^s = \underline{\hat{\xi}\, p^s}$$

$$= \begin{bmatrix} & -\omega \\ \omega & \end{bmatrix} p^s$$

$$= \begin{bmatrix} & -\omega \\ \omega & \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$= \omega \begin{bmatrix} -y \\ x \end{bmatrix}$$

$$\boxed{= \omega\, p^{s\perp}}$$

$$\boxed{v^s \bar{n} \geq 0}$$

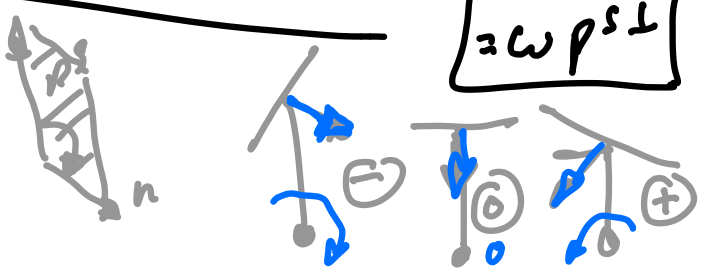
$$\hat{\xi} v^s \bar{n} \geq 0$$

$$\omega\, \underbrace{p^{s\perp} \bar{n}} \geq 0$$

$$\omega \cdot m \geq 0$$

$$\underbrace{p^s \times \bar{n}}$$

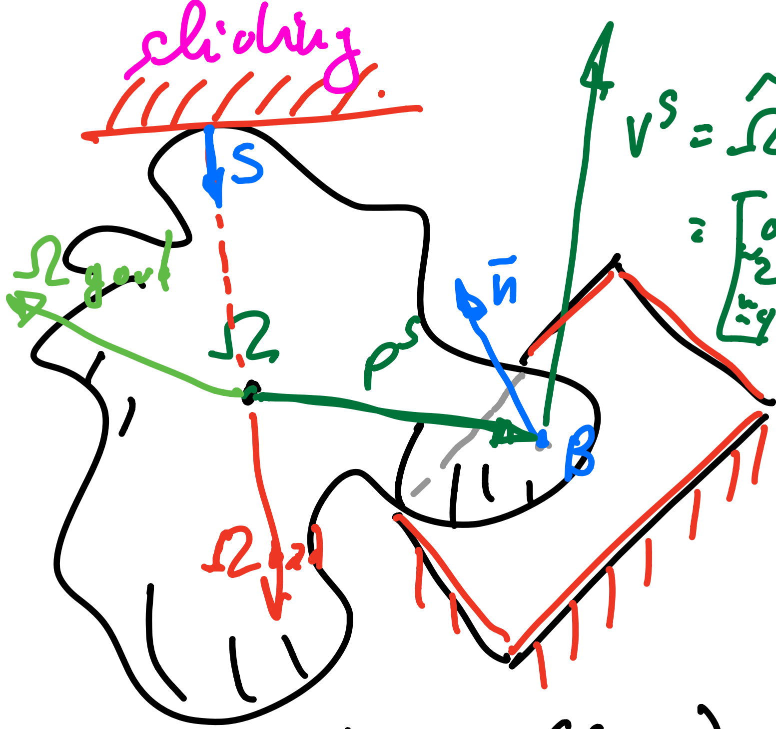
$m < 0$



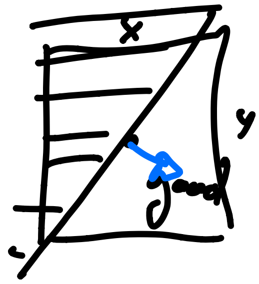
DoF ω = 1 DoF only half of [illegible]

$\omega \in \mathbb{R}_0^+$

② SO(3)



$$v^s = \hat\Omega p^s = \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$



$v^s \bar n \geq 0$

$\hat\Omega p^s \bar n \geq 0$

$(\Omega \times p^s) \bar n \geq 0$

$\Omega (p^s \times \bar n) \geq 0$

$\Omega\, m \geq 0$

$(a \times b) c = a (b \times c)$

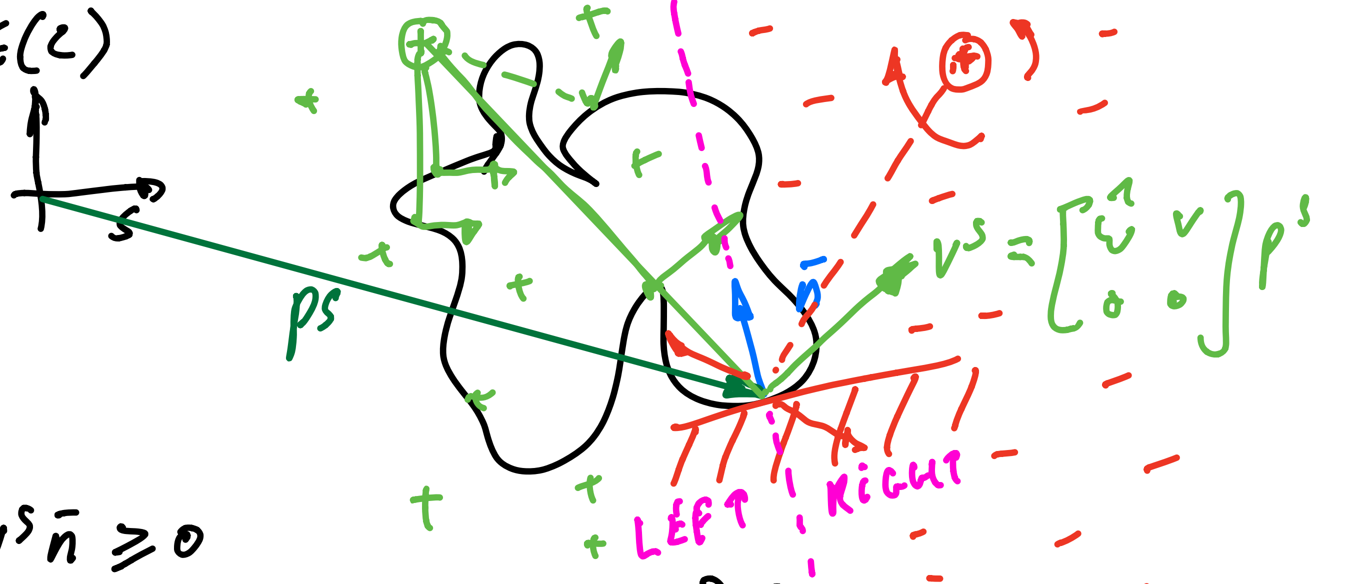
<u>DOF?</u>

3 DOF? "half"

2 DOF direction

1 DOF magn.

③ SE(2)



$V^s \bar n \geq 0$

$\hat\xi p^s \bar n \geq 0 \longleftrightarrow (\omega p^{s\perp} + v)\bar n \geq 0$

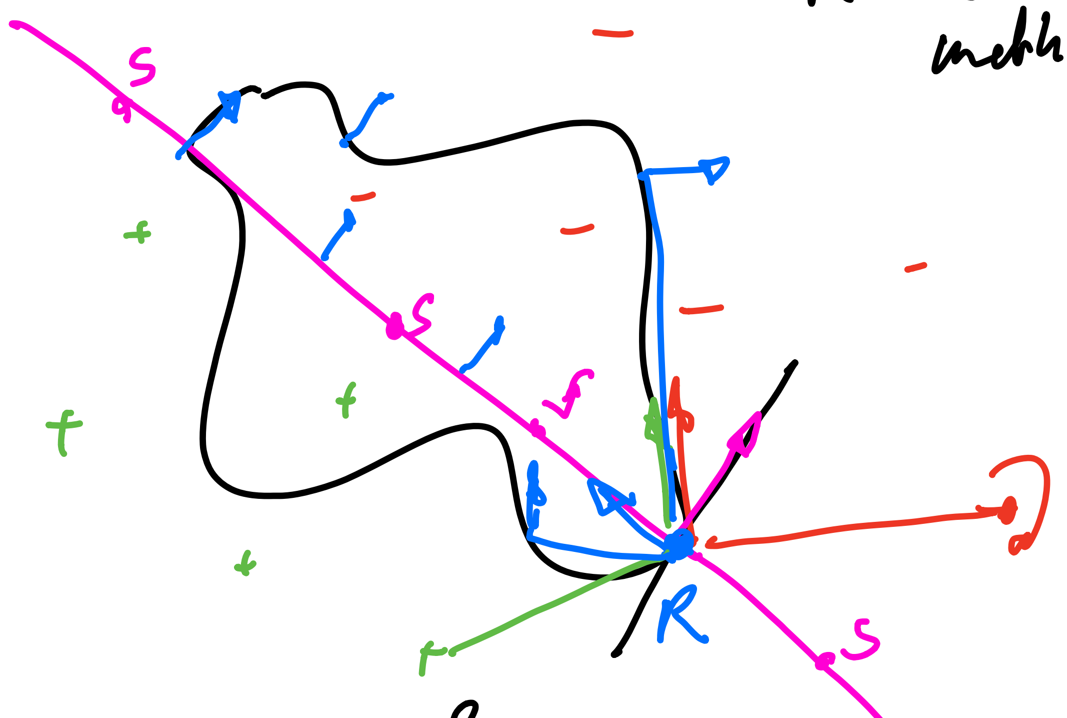
$\begin{bmatrix}\hat\omega & v\\ 0 & 0\end{bmatrix}\begin{bmatrix}x\\ y\\ 1\end{bmatrix} \cdot \bar n \geq 0$

$\begin{bmatrix} m = p^{s\perp}\bar n \\ \bar n \end{bmatrix}$

SE(2) $\underbrace{(m, \bar n)}_{3} \cdot \underbrace{\overline{(\overset{1}{\omega}, \overset{2}{v})}}_{3} \geq 0$

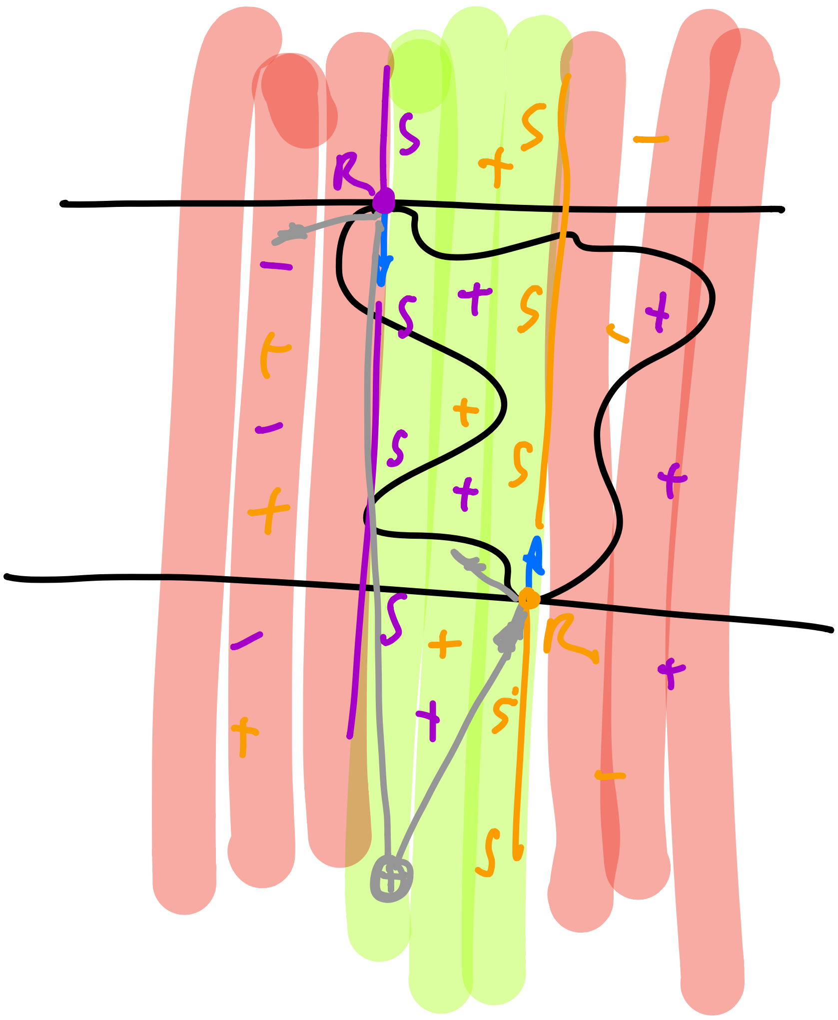
SO(3) $\underset{3}{\bar n} \cdot \underset{3}{\underline{\Omega}} \geq 0$

Reuleaux's method



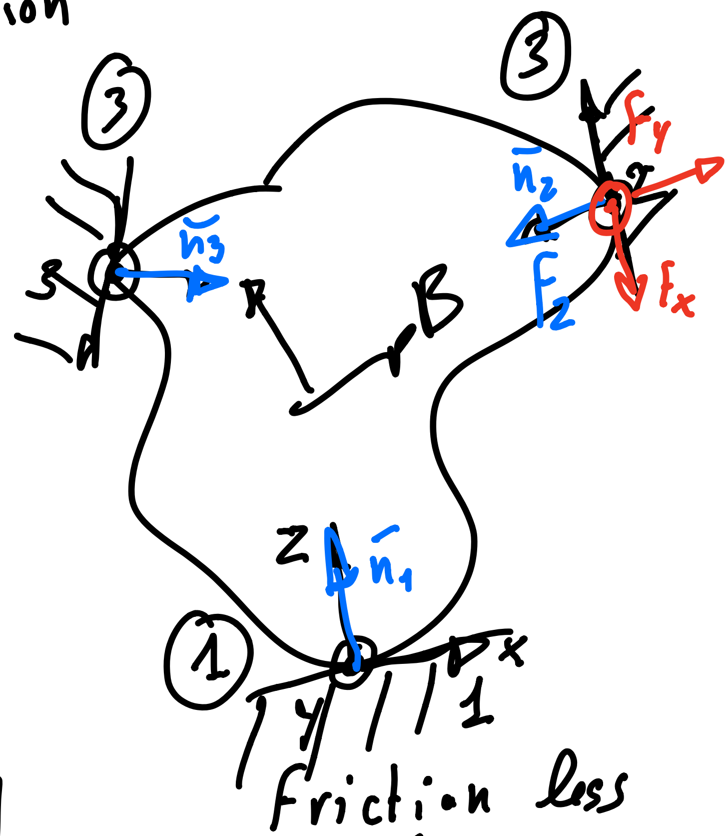
a) IRC on the line ?

b) IRC on the contact point ?



(4) SE(3) $\underbrace{(m, \bar{n})}_{\text{plücker 6dof}}^{T} \underbrace{(\ell, v)}_{V \text{ 6dof}} \geq 0$

⑤ Friction



contact w. Coulomb Friction

FC_2

$$\boxed{\begin{aligned}&\sqrt{f_x^2+f_y^2} \le \mu f_z\\ &f_z \ge 0\end{aligned}}$$

$$F_2 = \begin{bmatrix}0\\0\\0\\f\end{bmatrix} : \underbrace{\begin{bmatrix}0\\ \hline I\end{bmatrix}}_{B_2} f_{3D}$$

F_B can the contacts withstand?

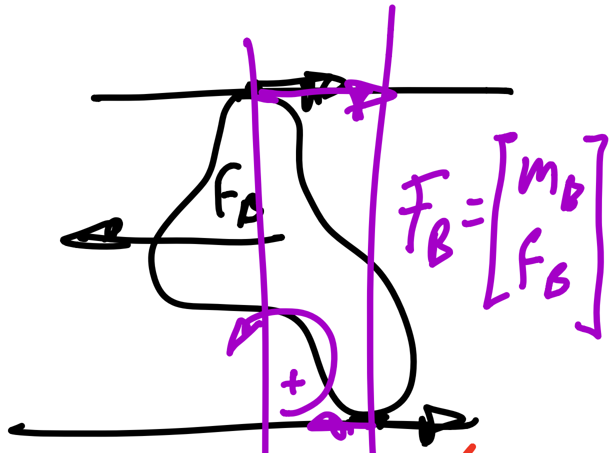
$$F_1 = \begin{bmatrix}0\\0\\0\\ \hline 0\\0\\1\end{bmatrix} f_z \qquad \boxed{f_z \ge 0}$$

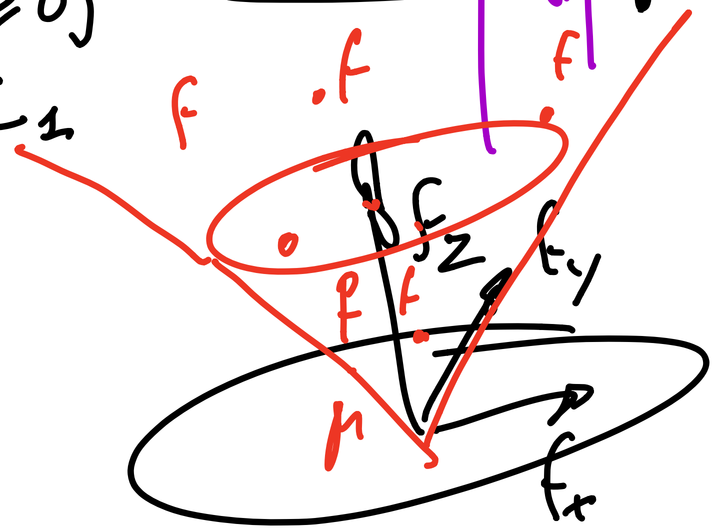
FC_1

$$\underline{F_1 = B_1 f_z}$$

$$f_z = \{f_z \ge 0\}$$

FC_1





$$F_B = [Ad_{1T_B}]^T F_1 :$$
$$[Ad_{2T_B}]^T F_2 : \sum [Ad_{iT_B}]^T F_i$$
$$[Ad_{3T_B}]^T F_3$$

$$= \sum \underbrace{[Ad_i T_b]^T B_i}_{G_i} f_i \qquad f_i \in FC_i$$

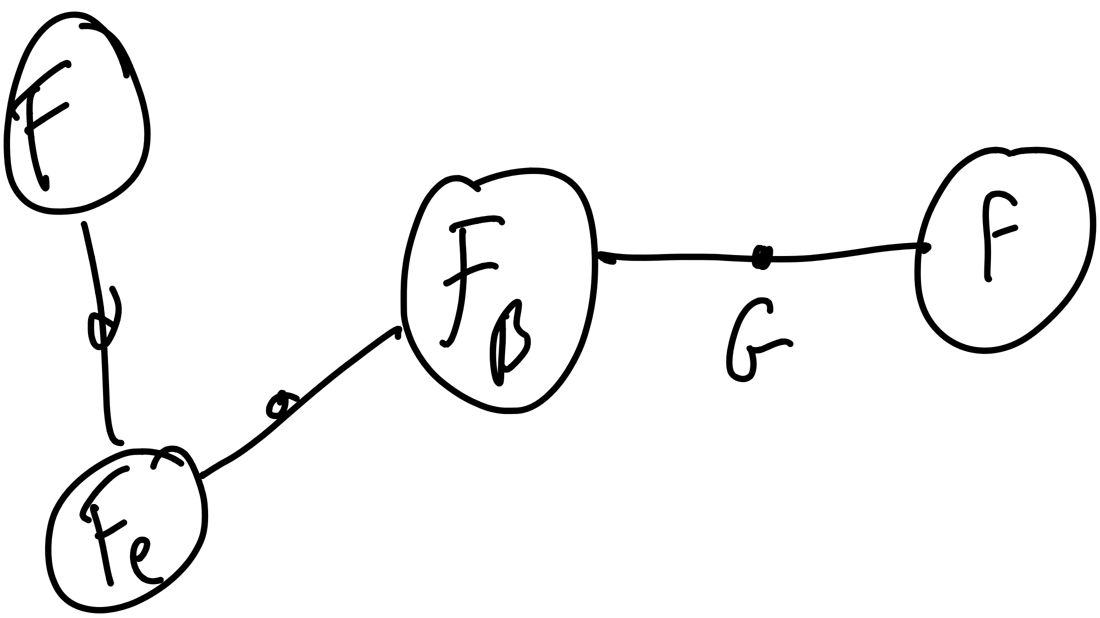
$$F_B = \sum G_i f_i \qquad f_i \in FC_i$$

$$F_B = G F \qquad F \in \mathbb{R}^7$$

$$\boxed{F \in FC} = \bigcup_i FC_i$$

$$\left[f \right] = \left[6\times 7\ G \right] \left[F \right]$$

3 min.



$F \in \mathbb{R}^7$

$\boxed{F \in FC}$